Chapter16 Instrumental variable estimation

SeokHoon Park

August 12, 2021

Seoul National University

Table of Contents

Table of Contents

Introduction

- The causal inference methods described so far in this book rely on a key untestable assumption: all variables needed to adjust for confounding and selection bias have been identified and correctly measured
- Instrumental method(IV method): do not require all adjustment factors

Table of Contents

The Three instrumental conditions for IV method

IV method need an variable Z that satisfies

(i) Z is associated with A
(ii) Z does not affect Y except through its potential effect on A
(iii) Z and Y do not share causes

Z is an instrumental variable
In the double-blind randomized trial, Z variable meets (i),(ii), and (iii)

The Three instrumental conditions for IV method

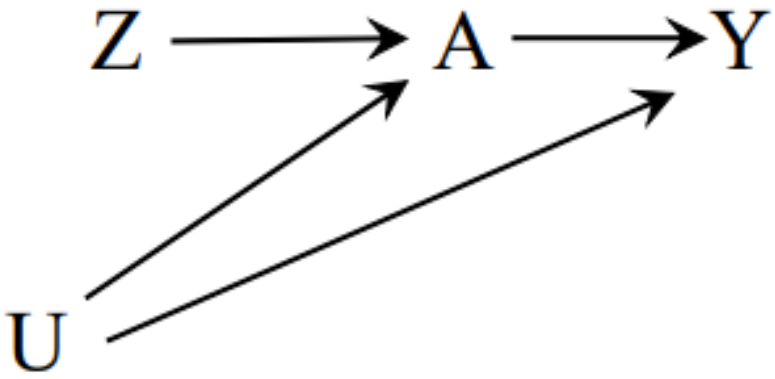

Figure 16.1

- Z : Instrumental variable (1: treatment , 0:placebo)
- A : treatment
- U : confounder

The limitation of three conditions

- Of the three instrumental conditions, only condition (i) is empirically verifiable
- We can only assume that conditions (ii) and (iii) hold
- We refer to Z as a proposed or candidate instrument

Table of Contents

A fourth identifying condition: homogeneity

- Usual IV estimand = $\frac{E(Y|Z=1)-E(Y|Z=0)}{E(A|Z=1)-E(A|Z=0)}$
- The three instrumental conditions (i)-(iii) are insufficient to ensure that IV estimand is the average causal effect of treatment A on Y
- So, a fourth condition (iv) *effect homogenity* is needed

A fourth identifying condition: homogeneity

- version 1: Constant effect of treatment A on outcome Y
- version 2: $E(Y^{a=1} - Y^{a=0}|Z = 1, A = a) = E(Y^{a=1} - Y^{a=0}|Z = 0, A = a), a = 0, 1$
- version 3: $E(Y^{a=1}|U) - E(Y^{a=0}|U) = E(Y^{a=1}) - E(Y^{a=0})$
- version 4: $E(A|Z = 1, U) - E(A|Z = 0, U) = E(A|Z = 1) - E(A|Z = 0)$

Table of Contents

The usual IV estimand

- Under condition (i)-(iv) ,
  $E(Y^{a=1}) - E(Y^{a=0}) = \frac{E(Y|Z=1)-E(Y|Z=0)}{E(A|Z=1)-E(A|Z=0)}$

proof : Technical point 16.3

The usual IV estimand

- Usual IV estimand $= \frac{E(Y|Z=1) - E(Y|Z=0)}{E(A|Z=1) - E(A|Z=0)}$
- Standard IV estimator $= \frac{E(Y|\hat{Z}=1) - E(Y|\hat{Z}=0)}{E(A|\hat{Z}=1) - E(A|\hat{Z}=0)}$

The usual IV estimand

1. We estimated the numerator an denominator of the IV estimand by simply calculating the four sample averages

2. $E(A|Z) = \alpha_0 + \alpha_1 Z$ $\mathrm{E(Y|Z)}=\beta_0 + \beta_1 Z$

3. Another method is using linear model to estimate standard IV estimator : the $two - stage - least - squares\ estimator$
   1. $E(A|Z) = \alpha_0 + \alpha_1 Z$
   2. $E(Y|Z) = \beta_0 + \beta_1 E(\hat{A}|Z)$
   3. $\hat{\beta}_1$: standard IV estimator

Table of Contents

An alternative fourth condition: monotonicty

- *homogenity* condition is not implausibility in many settings
- We use an alternative condition

An alternative fourth condition: monotonicty

- If we knew the values of the two counterfactual treatment variables $A^{z=1}, A^{z=0}$ for each individual
- we could classify all individuals in the study population into four disjoint subpopulations
  1. $Always - takers : A^{z=1} = 1, A^{z=0} = 1$
  2. $Never - takers : A^{z=1} = 0, A^{z=0} = 0$
  3. $Compliers : A^{z=1} = 1, A^{z=0} = 0$
  4. $Defiers : A^{z=1} = 0, A^{z=0} = 1$

An alternative fourth condition: monotonicty

- When no *defiers* exist, we say that there is monotonicity
- $A^{z=1} \geq A^{z=0}$
- Let us replace any of the *homogenity* conditions by the monotonicity condition
- Then $E(Y^{a=1}) - E(Y^{a=0}) \neq \frac{E(Y|Z=1)-E(Y|Z=0)}{E(A|Z=1)-E(A|Z=0)}$
- $E(Y^{a=1} - Y^{a=0}|A^{z=1} = 1, A^{z=0} = 0) = \frac{E(Y|Z=1)-E(Y|Z=0)}{E(A|Z=1)-E(A|Z=0)}$

An alternative fourth condition: monotonicty

- While homogeneity is often an implausible condition, monotonicity condition appeared credible in many settings
- IV method under monotonicity cannot identify the average causal effect in the population, only in the subpopulation of compliers, but that seemd a price worth paying in order to keep powerful IV methods in our toolbox

An alternative fourth condition: monotonicty

- The estimation of the average causal effect of treatment in the compliers under monotonicty has been criticized on several grounds
- If the effect in the compliers is considered to be of interest, relying on monotonicity seems a promising approach
- However, caution is needed when using this approach in more complex settings and observational studies